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A Definition of Time

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Abstract We present a definition of time based on a particle's interaction with the Higgs field. Just as a particle acquires mass by interacting with the Higgs field our model proposes that time is acquired via the energy of virtual particles participating in the quantum exchange interactions with Higgs particles. We show that, for macroscopic time, this definition accords with the Lorentz transformation of special relativity.

Keywords Planck scale · Planck-Einstein equation · Doubly-special relativity · Higgs field time observer · Time measurement · Special relativity

1 Introduction

String theory and loop quantum gravity theory claim that space and time are ultimately discrete. In spite of this, however, there has not been a serious attempt to derive the continuum equations of General and Special Relativity from a discrete space and time perspective. The main objective of this note is to present a definition of time based on the fundamental Planck scale that, on macroscopic scales, is consistent with the Lorentz transformation of Special Relativity. The definition of time is based on the energy of virtual particle's participating in a test particle's interaction with the Higgs field. Just as a particle acquires mass by interacting with Higgs particles we suggest that time is also acquired and measured by means of interactions with the Higgs field. We consider time to be an attribute of space, measured at a spatial location of Planck length by the amount of energy exchanged via very high frequency virtual particles. For macroscopic time, we show that the definition is in accord with the Lorentz transformation of special relativity.

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2 Definition of Time

We assume that space has a fundamental length scale, the Planck length, L_P, and that this scale is observer independent as in doubly-special relativity [1–3]. That is, any moving frame measures L_P as the smallest unit of length. If this were not the case then length contraction would contravene the notion of minimum length. With velocity limited by the speed of light, we have a minimum time unit, T_p. Since time dilation can only increase T_p, no assumption is necessary on a frame independent fundamental scale for time. We now state

Postulate 1 The Planck length is the smallest unit of length and this length is frame independent.

It is well known from the Standard Model that the vacuum in which all interactions take place is filled with Higgs particles. Quarks, Leptons, and W and Z bosons continuously interact with these particles as they move through the vacuum. The Higgs Field acts like molasses, slowing down particles that interact with it, thereby endowing them with mass. Virtual particles participating in the quantum exchange interactions with Higgs particles are manifested on all possible energy and time scales, even on the Planck scale. At any location O of Planck length we consider the virtual particles which exist for a Planck time, T_P, and hence are localized in a length L_P with energy E_P. We refer to these as maximal virtual particles (MVP). As a test particle interacts with the Higgs Field, MVP's come in and out of existence every T_P units of time. Since each MVP is characterized by a pulse of energy E_P and defines an amount of time T_P, we shall use the energy pulses of MVPs to define and measure time.

The Time-Energy Uncertainty relation, $\nabla E\ \nabla t \sim h$ for a virtual particle is interpreted as follows: ∇t is the lifetime of the particle, while ∇E is its energy during its lifetime [4]. For an MVP $\nabla t = T_P$ and $\nabla E = E_P$ and since it travels with velocity c it is confined to L_P. We can also view the MVPs, coming in and out of existence at O, as forming a wave with frequency $1/T_P \simeq (1/h)E_P$. At any location with Planck length in a stationary frame S, a pulse of energy E_P defines a time duration T_P. N pulses of energy E_P define a time duration NT_P.

Postulate 2 At any location of Planck length, the energy of a MVP defines an amount of time equal to its duration time, T_P.

Definition The time duration of an event measured at a location of Planck length is the total number of MVP energy pulses during the event multiplied by T_P.

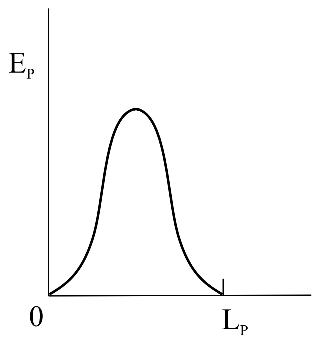


Fig. 1 Energy pulse associate with a maximal virtual particle

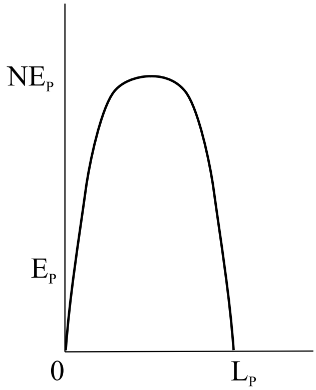


Fig. 2 An event of duration NT_P at $[0, L_P]$ interpreted as the energy of N maximal virtual particles

We may summarize the foregoing model with the following audio analogue. Suppose an observer is located at any location having Planck length. All around him there are pop sounds of all possible intensities. When the observer hears the loudest pop (corresponding to the energy pulse of an MVP) he knows a duration of time T_p has occurred.

3 Special Relativity

Our objective is to show that the foregoing definition of time, when extended to macroscopic scales, satisfies the Lorentz transformation. We consider a frame S' moving with velocity v with respect to a stationary frame S. Let I' denote the location of the ith Planck length in the S' frame. When the event commences both frames are aligned and I' corresponds to the ith Planck length in the S frame. In the S' frame an event resulting in the participation of a single MVP measures an amount of time $T'_P = L'_P/c$. The same single MVP event observed in the S' frame is measured by the MVP travelling toward it with relative velocity $c - v$. Thus, the passage of the MVP through I in the S frame takes an amount of time $L'_P/(c - v)$. Since $L_P = L'_P$, by Postulate 1, and $L'_P = T'_P c$, the time for the MVP to traverse I in the S frame is $T'_P c/(c - v)$. If we let $x = v/c$, we obtain the relation between time observation in the S and S' frames at the same location I of Planck length as:

$$T_p = T'_P/(1 - x) \tag{1}$$

We now consider how the observer, stationed at $[0, L_P]$ in the S frame, measures the time of the single MVP event at I, the ith Planck length location in the S frame? It is reasonable to assume that the energy associated with the MVP event at I is reduced and the degree of reduction depends on the location on the x-axis specified by i; the further away from $[0, L_P]$, the less energy will be measured at $[0, L_P]$, and hence less time. To model this effect mathematically we need a function acting on T_p that reduces it according to its location on the S frame. To that end let iL_P denote the ith Planck length in both frames when the two frames are aligned at their respective origins. Although the ith Planck length has the same length in both frames, the time duration of a MVP's passage through them are different; in the S' frame it is T'_P while in the S frame it is $T_p = T'_P/(1 - x)$. We shall model this effect by an operator A which, among other properties, satisfies $A(T'_P/(1 - x)) < T'_P/(1 - x)$. To reflect reduction of energy from the ith Planck length to $[0, L_P]$ we use the ith iterate of A, A^i. Let us now consider an event that spans N MVPs as depicted in Fig. 3. Since

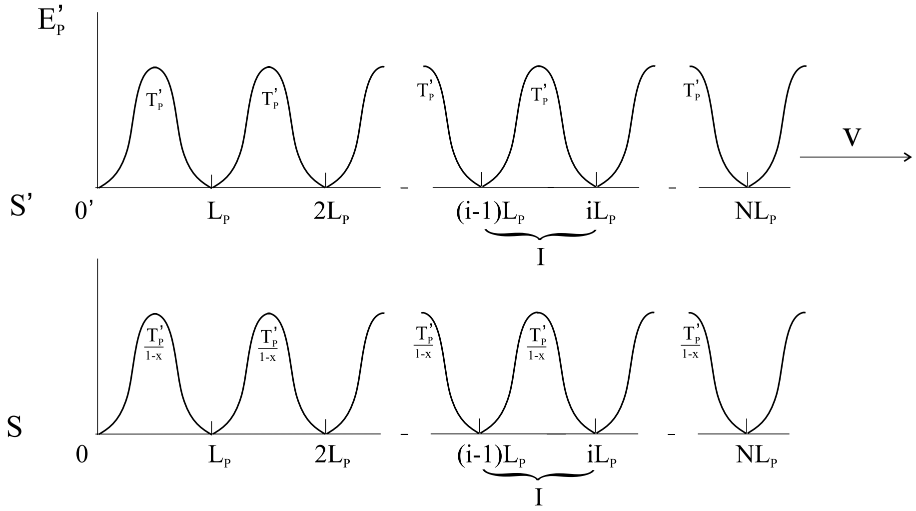


Fig. 3 An event of N MVP energy pulses as viewed in the moving frame S' and in the stationary frame S

we know an important property of time on macroscopic scales ($N \to \infty$), namely Lorentz invariance, it is of interest to see if our definition of time satisfies the Lorentz transformation as $N \to \infty$.

Let $C(0,1)$ denote the space of continuous functions on $(0,1)$. Note the function $1/(1-x)$, which is continuous on $(0,1)$ defines the time dilation due to one MVP and the relative motion between two frames at the same location. Let $x = v/c$. We now define an operator A with the following properties:

1) For any function f in $C(0,1)$, with $f > 0$ and $f(0) = 1$, the iterate $A^i f$ represents the amount of MVP energy transferred from the ith Planck length location in S to $[0, L_P]$ in S.
2) The total measured time at the origin of S, for large scale times, ($N \to \infty$), must satisfy the Lorentz time dilation transformation. That is, if T' is the proper macroscopic time in the S' frame, then the time in the S frame is given by $T' \frac{1}{\sqrt{1-x^2}}$. This implies that $f^*(x) = \frac{1}{\sqrt{1-x^2}}$ should be a fixed point of A.
3) We want f^* to be a stable fixed point of A. That is, f^* should be an attractor in $C(0,1)$ so that the approximations in using Planck scale lengths are valid.

Let $A : C(0,1) \to C(0,1)$ be defined by

$$Af(x) = \frac{1 + f^2(x) \cdot x^2}{f(x)}.$$

It is easy to very that f^* is a fixed point of A. It is our intention to prove that A satisfies all the above desired properties and that the iterates of $A^n f$, for certain f, converge to f^*.

Proposition 1 *For any function $f(x) > 0$ with $f(0) = 1$ the iterations $A^n f$ converge to the function*

$$f^*(x) = \frac{1}{\sqrt{1-x^2}}.$$

The proof is a consequence of the following three lemmas. Let

$$A_x(a) = \frac{1 + a^2x^2}{a}, \quad 0 < x < 1,\ a > 0.$$

Lemma 2 a) *If* $a > \frac{1}{\sqrt{1-x^2}}$, *then for those* a *satisfying this inequality we have* $A_x(a) < a$ *and* $A_x^2(a) < a$.

b) *If* $a < \frac{1}{\sqrt{1-x^2}}$, *then for those* a *satisfying this inequality we have* $A_x(a) > a$ *and* $A_x^2(a) > a$.

Proof Let $a > \frac{1}{\sqrt{1-x^2}}$. Then, $a^2 > \frac{1}{1-x^2}$, or $a^2 - a^2x^2 > 1$, and $a > \frac{1+a^2x^2}{a}$. To prove the second statement we continue. We have $a^2x^2 > (\frac{1+a^2x^2}{a})^2x^2$, or $(1+a^2x^2)\cdot\frac{a}{a} > 1 + (\frac{1+a^2x^2}{a})^2x^2$, which means $a > A_x^2(a)$.

The proof of the statement b) is similar. □

Lemma 3 *Let* $g(x) = \frac{\sqrt{1-x^2}}{x^2}$. *For* $x < 1/\sqrt{2}$ *we have* $f^*(x) < g(x)$ *and for* $x > 1/\sqrt{2}$ *we have* $f^*(x) > g(x)$. *For* a *between the graphs of* f^* *and* g *we have* $A_x(a) \le f^*(x)$ *and for the remaining* a *we have* $A_x(a) > f^*(x)$.

In particular:

a) *If* $a < f^*(x)$ *and* $x < 1/\sqrt{2}$, *then* $A_x(a) > f^*(x)$.
b) *If* $a > f^*(x)$ *and* $x > 1/\sqrt{2}$, *then* $A_x(a) > f^*(x)$.

Proof This follows by solving the inequality: $A_x(a) > \frac{1}{\sqrt{1-x^2}}$. □

Lemma 4 *If* $a > 0$, *then the sequence* $\{A_x^n(a)\}_{n\ge 0}$, $0 < x < 1$, *converges to* $f^*(x) = \frac{1}{\sqrt{1-x^2}}$.

Proof First we consider $a > f^*(x)$.

Let us assume $x < 1/\sqrt{2}$. If $a > g(x)$ then the sequence $A_x^n(a)$ decreases until it goes to or below $g(x)$ at some step n_0. If $A_x^{n0}(a) = g(x)$ then the next element $A_x^{n0+1}(a) = f^*(x)$ and all the following elements have the same value. If $A_x^{n0}(a) < g(x)$, then the following elements of the sequence oscillate below and above the value $f^*(x)$. By Lemma 2a) the elements above $f^*(x)$ converge to this value monotonically. The "below" elements of the sequence also converge to the same limit since A_x is continuous.

For $x > 1/\sqrt{2}$ the sequence $A_x^n(a)$ is decreasing and converges to $f^*(x)$ monotonically.

Now, let $a < f^*(x)$. If $x < 1/\sqrt{2}$, then $A_x(a) > f^*(x)$ and we have convergence by the first part of the proof.

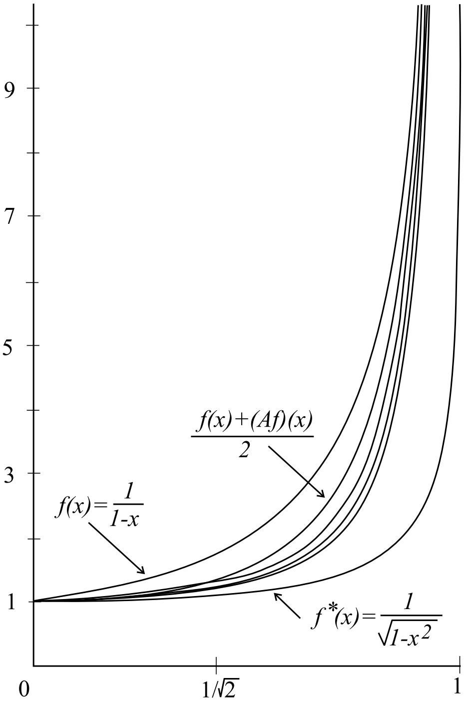
Let $x > 1/\sqrt{2}$. If $a \le g(x)$, then again $A_x(a) > f^*(x)$ and we have convergence by the first part of the proof. If $a > g(x)$, then the sequence $A_x^n(a)$ is increasing and converges to $f^*(x)$ monotonically. □

The following theorem is the consequence of Proposition 1.

Theorem 5 *Let* $f(x) = \frac{1}{1-x}$. *We have the convergence of the averages*

$$\frac{1}{N}\left(f + A(f) + A^2(f) + \cdots + A^{N-1}(f)\right) \to f^*.$$

Fig. 4 Averages $\frac{1}{N}\sum_{n=0}^{N-1} A^n(f)$, $N = 1, 2, 3, 4, 5$, for $f(x) = \frac{1}{1-x}$ (*top curve*) and the limit function $f^*(x) = \frac{1}{\sqrt{1-x^2}}$ (*bottom curve*)



This means that

$$\frac{T'_P}{N}[1/(1-x) + A^1(1/(1-x)) + A^2(1/(1-x)) + \cdots + A^{N-1}(1/(1-x))] \to \frac{T'_P}{\sqrt{1-x^2}}$$

or

$$T'_P[1/(1-x) + A^1(1/(1-x)) + A^2(1/(1-x)) + \cdots + A^{N-1}(1/(1-x))] \to \frac{NT'_P}{\sqrt{1-x^2}}$$

that is, the total time observed at $[0, L_P]$ in S approaches $\frac{NT'_P}{\sqrt{1-x^2}}$ as the number of MVPs increases to ∞. But NT'_P is the proper time observed in the S' frame. Hence we have derived the Lorentz transformation for time dilation.

It is of interest to know how the measured dilation times approaches the Lorentz transformation. The following lemma shows that at all scales the dilation function stays above the graph of $\frac{1}{\sqrt{1-x^2}}$.

Proposition 6 *If $f \geq f^*$, then the sequence of averages stays above the limit function f^*:*

$$\frac{1}{N}\sum_{n=0}^{N-1} A^n(f) \geq f^*.$$

Proof As we showed in Lemma 2, for $x > 1/\sqrt{2}$ and $a > f^*(x)$ the sequence $A^n_x(a)$ decreases monotonically and is above $f^*(x)$. The statement of the lemma follows.

For $x < 1/\sqrt{2}$ and $a > f^*(x)$ the sequence $A_x^n(a)$ decreases monotonically for some time and then starts to oscillate below and above $f^*(x)$. We will prove that

$$a > f^*(x) \quad \text{and} \quad A_x(a) < f^*(x) \quad \Longrightarrow \quad f^*(x) - A_x(a) < a - f^*(x).$$

This implies the statement of the lemma.

We want to show

$$\frac{1}{\sqrt{1-x^2}} - \frac{1+a^2x^2}{a} < a - \frac{1}{\sqrt{1-x^2}},$$

or

$$a^2(1+x^2) - a\frac{2}{\sqrt{1-x^2}} + 1 > 0.$$

Standard calculations show that this holds for $a > \frac{1}{\sqrt{1-x^2}}$. □

Note There may be other operators than A that reflect energy decay over distance and have properties such as A much as on [0, 1] all contraction operators will cause the desired reduction and have 0 as the fixed point.

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